2007 VCAA Further Mathematics Exam 1 Solutions
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SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
A	B	A	D	C	C	C	D	D	A	E	C	E

SECTION B
Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
B	C	B	C	E	B	E	D	A

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
A	B	B	A	B	D	C	D	E

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
D	C	E	B	D	E	C	B	A

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
A	C	C	D	E	D	E	D	A

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
E	C	A	B	D	E	C	B	B

Module 6: Matrices

1	2	3	4	5	6	7	8	9
A	D	B	B	A	D	E	B	C

SECTION A Core: Data analysis

Q1 A

Q2 B

Q3 $z = \dfrac{56 - 67}{10.2} = -1.08$ A

Q4 D

Q5 $14 = Q_U$, C

Q6 Jellyfish from location A have a lower median and a larger range. C

Q7 $r^2 = 0.9034^2 = 0.816 = 81.6\%$ C

Q8 $\text{Gradient} = 0.87 = \dfrac{0.87}{1}$ D

Q9 $\text{Gradient} \approx \dfrac{\log 869 - \log 100}{9 - 5} = 0.23$ D

Q10 The categorical data are converted to numerical. A

Q11 E

Q12 The two median points are $(6.5, 3000)$ and $(30.5, 7000)$.

$\text{Slope} = \dfrac{7000 - 3000}{30.5 - 6.5} = 167$ C

Q13 $\text{Average of data} = \dfrac{Sum}{12} = \dfrac{43872}{12} = 3656$.

$\text{Seasonal index for September} = \dfrac{4597}{3656} \approx 1.26$ E

SECTION B
Module 1: Number patterns and applications

Q1 $r = \dfrac{6}{24} = 0.25$ B

Q2 $d = \dfrac{29 - 15}{2} = 7$, $\therefore t_4 = t_3 + 7 = 29 + 7 = 36$ C

Q3 $t_2 = at_1 + 6$, $\therefore 21 = 5a + 6$, $a = 3$ B

Q4 $680 \times \left(1 - \dfrac{4}{100}\right)^5 = 554$ C

Q5 Geometric, $r = 1 - 0.04 = 0.96$, $\therefore \dfrac{W_{n+1}}{W_n} = 0.96$,

i.e. $W_{n+1} = 0.96 W_n$ E

Q6 Arithmetic series, $S_n = \dfrac{n}{2}(a + l) = \dfrac{20}{2}(1 + 20) = 210$ B

Q7 E

Q8 D

Q9 Geometric series: 15 + 13.5 + 12.15 + …

$a = 15$, $r = \dfrac{13.5}{15} = 0.9$, $S_{14} = \dfrac{15(1 - 0.9^{14})}{1 - 0.9} \approx 116$ A

Module 2: Geometry and trigonometry

Q1 $\cos\theta° = \dfrac{A}{H} = \dfrac{6}{10}$ A

Q2 $\text{Height} = 2200 \tan 37° \approx 1658 \text{ m}$ B

Q3 $\text{Side length} = \dfrac{24.50}{2} - 3.79 = 8.46 \text{ m}$

$\text{Length of diagonal} = \sqrt{3.79^2 + 8.46^2} \approx 9.3 \text{ m}$ B

Q4 $V = A \times l = 0.048 \times 12 = 0.576 \text{ m}^3$ A

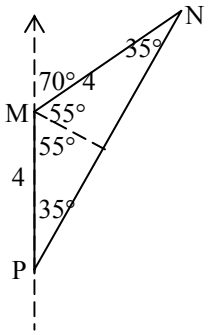
Q5 $10\ \text{cm}^2 : 4000\ \text{m}^2$
$\therefore 1\ \text{cm}^2 : 400\ \text{m}^2$
$\therefore 1\ \text{cm} : 20\ \text{m}$ B

Q6 Hemisphere: $\frac{1}{2} \times 4\pi r^2 = 2\pi(20^2) = 2513.3$
Curve side surface: $\pi \times d \times h = \pi(40)(30) = 3769.9$
Circular base: $\pi r^2 = \pi(20^2) = 1256.6$
TSA $= 2513.3 + 3769.9 + 1256.6 \approx 7540\ \text{cm}^2$ D

Q7 Max length $= 36\sqrt{1^2 + 1^2 + 1^2} = 36\sqrt{3} \approx 62\ \text{cm}$ C

Q8 D

Q9 All three calculations give the same length for PN. E



Module 3: Graphs and relations

Q1 D

Q2 When $n = 8$, $C = 60 + 55 \times 8 = 500$. C

Q3 E

Q4 $1 \times n = 60 + 0.20n$, $0.8n = 60$, $n = 75$ B

Q5 1.5 hours and 3 hours, \$20 + \$25 = \$45 D

Q6 E

Q7 Let $y = kx^3$. From the given graph, when $x^3 = 3$, $y = 1$.
$\therefore k = \frac{1}{3}$. $\therefore y = \frac{1}{3}x^3$.
When $x = 1$, $y = \frac{1}{3}$. C

Q8 B, because the equations represent two parallel lines and there is no intersection (no solution).

Q9 The gradient of the objective function $= -\frac{2}{k} < \frac{0-50}{100-50}$,
i.e. $-\frac{2}{k} < -1$, $\frac{2}{k} > 1$, $k < 2$. A

Module 4: Business-related mathematics

Q1 $\frac{20}{500} = 0.04 = 4\%$ A

Q2 Depreciation $= 30000 - 8000 = 22000$
Number of km $= \frac{22000}{0.25} = 88000$ km C

Q3 $27000 = \frac{P(8)(6)}{100}$, $P = \$56250$ C

Q4 Amount $= 2560 + 0.06(200000 - 115000) = \$7660$ D

Q5 E

Q6 Amount $= 10000\left(1 + \frac{5}{100}\right)^{10} = 10000 \times 1.05^{10}$ D

Q7 A year ago, salary $= \frac{42000}{1.02} = 41176.47$.
Two years ago, salary $= \frac{41176.47}{1.03} \approx \$39977$ E

Q8 Balance $= 720 - 180 = 540$.
Interest on balance $= \frac{540 \times 12 \times 2}{100} = 129.60$
Monthly repayment $\frac{540 + 129.60}{24} = \$27.90$ D

Q9 Monthly instalment = \$1938.25 by TVM Solver
Total repayment $= 1938.25 \times 240 = \$465180$
Total interest $= 465180 - 250000 = \$215180$ A

Module 5: Networks and decision mathematics

Q1 E

Q2 $v + f = e + 2 = 12 + 2 = 14$ C

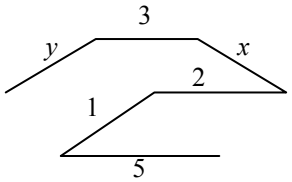
Q3 A

Q4 $4 + 1 + 3 + 2 = 10$ B

Q5 D

Q6 Critical path: CDFKL.
Earliest start time for L is $5 + 0 + 4 + 7 = 16$ E

Q7



$y + 3 + x + 2 + 1 + 5 = 19$, $\therefore x + y = 8$ C

Q8 Two-step dominance matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 & 2 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}\begin{array}{c} \\ 4 \\ 5 \\ 3 \\ 2 \\ 1 \end{array}$$

5 is the highest. B

Q9 In 2007, $4+3+2+1=10$.
In 2008, $6+5+4+3+2+1=21$.
$21-10=11$ extra games B

Module 6: Matrices

Q1 $\begin{bmatrix} 0+5 & -4+4 \\ 2-2 & 5+2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$ A

Q2 D

Q3 $A^{-1} = \dfrac{1}{8\times 3-5\times 4}\begin{bmatrix} 3 & -4 \\ -5 & 8 \end{bmatrix} = \dfrac{1}{4}\begin{bmatrix} 3 & -4 \\ -5 & 8 \end{bmatrix}$

$X = \dfrac{1}{4}\begin{bmatrix} 3 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} = \begin{bmatrix} -4.25 & -5.5 \\ 9.75 & 12.5 \end{bmatrix}$ B

Q4 B

Q5 A

Q6 $\begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.7 \end{bmatrix}^n \begin{bmatrix} 1568 \\ 1105 \\ 894 \end{bmatrix} = \begin{bmatrix} 1605.15 \\ 713.4 \\ 1248.45 \end{bmatrix}$ when n is large enough. D

Q7 Leading diagonal elements are all zeros, and sum of elements = 1 in each column. E

Q8 First: $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ Second: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Third: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Fourth: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Fifth: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Sixth: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore DBCAAA$ B

Q9 $M(NP) = \begin{bmatrix} 4 & 1 & 7 & 2 \\ 0 & 9 & 7 & 4 \\ 4 & 3 & 3 & 1 \end{bmatrix}$

Order: $(3\times 4)((4\times 5)(5\times 4)) = (3\times 4)$ C

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